Repetitive Control of Non-minimum Phase Systems Along B-spline Trajectories

Luigi Biagiotti, Federico Califano, Claudio Melchiorri

***Abstract*— In this paper, a novel repetitive control scheme is presented and discussed, based on the so called B-spline filters. This type of dynamic filters are able to provide a B-spline trajectory if they are fed with the sequence of proper control points that define the trajectory itself. Therefore, they are ideal tools for generating online the reference signal with the prescribed level of smoothness for driving dynamic systems, e.g. with a feedforward compensator. In particular, the so-called Continuous Zero Phase Error Tracking Controller (ZPETC) can be used for tracking control of non-minimum phase systems but because of its open-loop nature cannot guarantee robustness with respect to modelling errors and exogenous disturbances. For this reason, ZPETC and trajectory generator have been embedded in a repetitive control scheme that allows to nullify interpolation errors even in non-ideal conditions, provided that the desired reference trajectory and the disturbances are periodic. The asymptotic stability of the overall control scheme has been proved and its performances have been demonstrated by considering a well-known non-minimum phase plant, i.e. a flexible link arm.**

I. INTRODUCTION

Quite often, in industrial applications, the given tasks present a cyclic or repetitive nature; this means that, from a control perspective, the plant is required to track a periodic exogenous signal whose cycle time is supposed to be known in advance. When cyclic motions are considered, the Repetitive Control (RC) approach represents a quite standard and effective solution to achieve asymptotic perfect tracking, being able to cancel tracking errors over repetitions by learning from previous iterations, [1], [2], [3].

The idea of modifying a B-spline reference trajectory by applying a repetitive control scheme on the corresponding control points was proposed in [4]. Thanks to the possibility of generating B-spline trajectories by means of dynamic filters [5], the trajectory planner has been inserted in an external feedback control loop that modifies in real-time the control points of the B-spline curve so that the interpolation error at the desired via-points converges to zero. However, the proposed result relies on the hypothesis that the plant is already controlled and is characterized by an acceptable tracking capability within a certain frequency range, i.e.

$$G(j\omega) \approx 1, \ \ \omega < \omega_m$$

where ω_m denotes the maximum frequency of the reference input. Starting from this result, in this paper a novel repetitive control scheme is presented, in which the real-time modification of the control points that define the reference B-spline trajectory is combined with a feedforward compensation technique that can be applied also to non-minimum phase systems and allows removing the hypothesis on the dynamic behaviour of the plant. As a matter of fact, it is well-known that unstable zeros cannot be eliminated by feedback control and therefore they continue to affect the system even if it has been stabilized with a proper feedback law.

It will be shown that the B-spline trajectory generator and the feedforward controller, known as (Continuous) Zero Phase Error Tracking Controller (ZPETC), guarantee asymptotic stability of the Repetitive Control scheme for any (stable) continuous linear time invariant system and is robust with respect to both external and parametric errors.

The paper is organized as follows. In Sec. II a general overview of the ZPET controller in the continuous time formulation is given. Then in Sec. III the dynamic filters used to generate B-spline trajectories are shortly explained. In Sec. IV the RC scheme based on B-spline and ZPETC is presented and its properties are analysed. Finally, in Sec. V a typical example of non-minimum phase system, i.e. a one link flexible robot, is modelled and successfully controlled by means of the proposed scheme. Conclusions are reported in Sec. VI.

II. FEEDFORWARD CONTROL OF NON-MINIMUM PHASE SYSTEMS: THE ZPETC

The ZPETC is a feedforward control method proposed by Tomizuka [6] to improve tracking accuracy in motion control for non-minimum phase systems. It can be seen as an extension of the feedforward complete dynamic inversion of the system to be controlled in which the unstable zeros of the system are multiplied by their opposite as alternative to cancellation, that would be infeasible. Consider for instance a generic SISO LTI system $G(s)$, supposed stable[1], that can be modelled as

$$G(s) = \frac{N^s(s)N^u(s)}{D(s)}$$

where $D(s)$, $N^s(s)$ and $N^u(s)$ denote respectively the polynomials corresponding to the n poles, the m_s stable zeroes and m_u unstable zeroes of the plant. The ZPET

[1] If $G(s)$ is unstable, a stabilizing controller can be applied.

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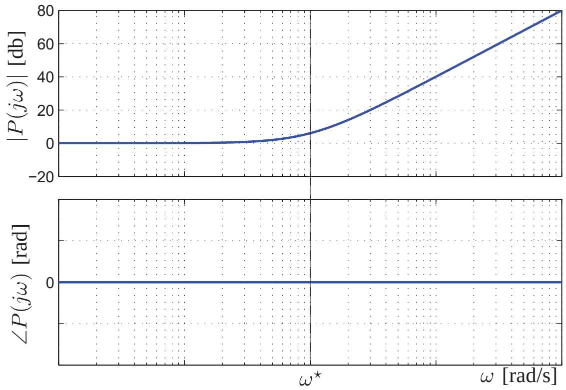

Fig. 1. Bode diagrams of the open-loop transfer function $P(s)$ obtained with a plant with an unstable real zero located at $\omega^\star$.

controller in its continuous-time version (see [7]) assumes the form

$$R(s) = \frac{D(s)N^u(-s)}{N^s(s)[N^u(0)]^2}. \tag{1}$$

Clearly, if $m_u = 0$, i.e. the system is minimum phase, $R(s)$ is a standard feedforward controller that completely cancels the dynamics of the plant ($R(s) = G^{-1}(s)$). In case $m_u \neq 0$, the cascade $P(s) = G(s)R(s)$ becomes

$$P(s) = \frac{N^u(-s)N^u(s)}{[N^u(0)]^2}. \tag{2}$$

Note that unstable zeros cannot be cancelled with a feedforward regulator because it would be unstable and cannot be modified with a feedback controller. The use of ZPETC represents a solution to this problem because of the following properties of the resulting open-loop function $P(s)$:

Property 1. $|P(j\omega)| \approx 1$ for $\omega < \omega^\star$, being $\omega^\star$ the break frequency corresponding to the smallest unstable zero. The asymptotic slope of the Bode plot for magnitude is $+2m_u \times 20$ db/decade.

Property 2. $\angle P(j\omega) = 0$ rad, $\forall \omega$, since $P(s)$, for any unstable zero, contains also its opposite, and therefore their phase contributions compensate each other.

In Fig. 1 the Bode diagrams of the open-loop transfer function $P(s)$ related to a plant with an unstable real zero located at $\omega^\star$ is shown.

A major problem of ZPETC, and in general of inversion-based feedforward approach, concerns the practical implementation of the controller, which is generally noncausal. In fact, the relative degree γ of a causal plant $G(s)$ cannot be negative, i.e. $\gamma \geq 0$. As a consequence the relative degree of $R(s)$, that is $\rho = m_s - (n + m_u) = -\gamma - 2m_u$ is usually negative. A way to cope with this problem is based on the precognition of the reference signal. By diving numerator and denominator, $R(s)$ can be rewritten as

$$R(s) = \sum_{i=0}^{|\rho|} \alpha_i s^i + W(s)$$

where $W(s)$ is a strictly proper transfer function. Consequently, the control action of the ZPETC will result

$$U(s) = \sum_{i=0}^{|\rho|} \alpha_i s^i Y_r(s) + W(s)Y_r(s) \tag{3}$$

where $Y_r(s)$ denote the Laplace transform of the reference input, and in time domain

$$u(t) = \sum_{i=0}^{|\rho|} \alpha_i \frac{d^i y_r(t)}{dt^i} + \mathcal{L}^{-1}\{W(s)Y_r(s)\}. \tag{4}$$

Therefore, in order to compute the control signal $u(t)$ the knowledge of $y_r(t)$ and its first $|\rho|$ derivatives is necessary. Obviously, in order to guarantee the feasibility of $u(t)$, the $|\rho|$ derivatives must be limited and consequently $y_r(t) \in C^{|\rho|-1}$.

III. Set-point generation via B-spline filters

In practical applications, smooth reference signals are defined by using spline functions interpolating a set of desired via-points $q_k^\star$, $i = 0, \ldots, n-1$. Uniform spline curves in the so-called B-form (i.e. B-splines characterized by an equally-spaced by T distribution of the knots) are defined, with the standard notation in [8], by

$$q_u(t) = \sum_{k=0}^{n-1} p_k\, B^d(t - kT), \qquad 0 \leq t \leq (n-1)T \tag{5}$$

As shown in [5], a B-spline trajectory of degree d can be generated by means of a chain of d dynamic filters defined as

$$M(s) = \frac{1 - e^{-sT}}{Ts}$$

fed by the staircase signal $p(t)$ obtained by maintaining the value of each control point p_k defining the curve for the entire period $kT \leq t < (k+1)T$, by means of a zero-order hold $H_0(s)$. The degree d of the spline and therefore the number of filters composing the B-spline generator of Fig. 2 determines the smoothness of the output trajectory since the resulting spline is a function of class C^{d-1}. Moreover, by means of the filter for B-spline generation it is possible to compute the profiles of all the derivatives of the trajectory up to the order d, as shown in Fig. 3.

Combining ZPETC with B-spline generator

The implementation of (4) by means of the B-spline generator of Fig. 3 is straightforward provided that the order of the B-spline meets the condition

$$d \geq |\rho| = \gamma + 2m_u \tag{6}$$

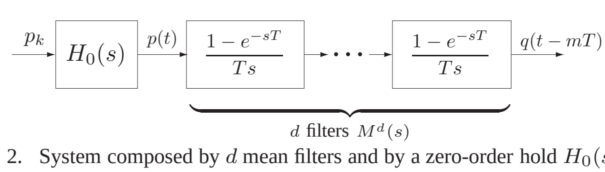

Fig. 2. System composed by d mean filters and by a zero-order hold $H_0(s)$ for the computation of continuous-time B-spline trajectories of degree d.

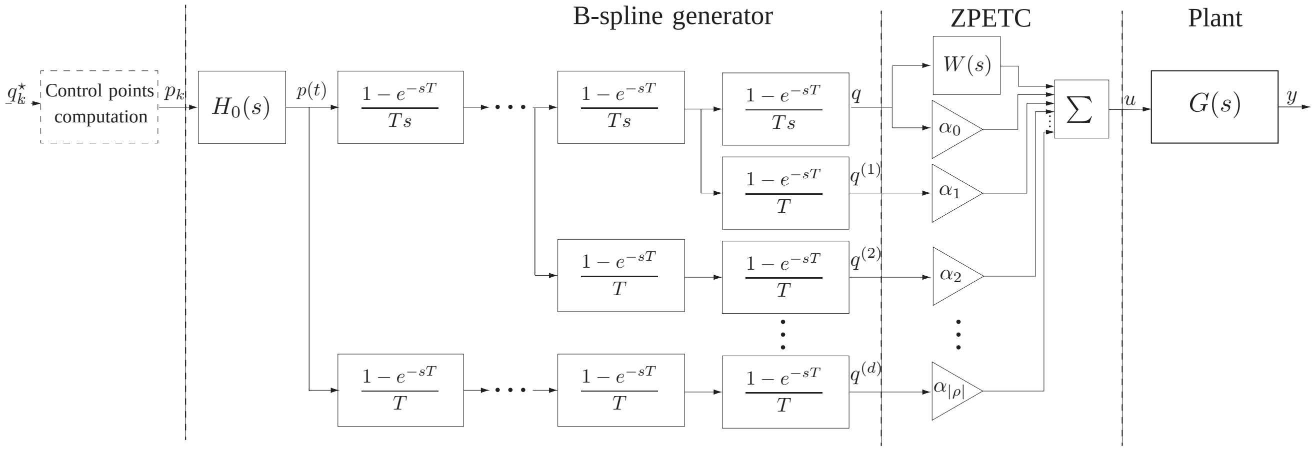

Fig. 3. Block-scheme representation of feedforward control based on B-spline trajectory and ZPETC ($d = |\rho|$) for the plant $G(s)$.

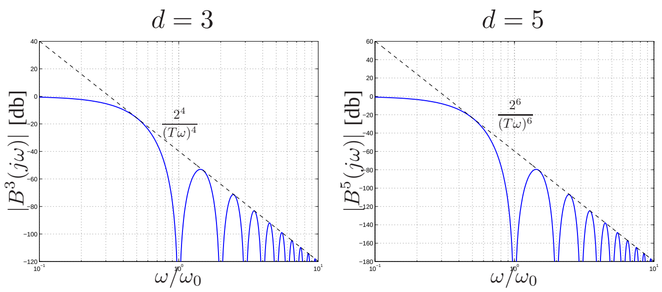

Fig. 4. Frequency spectrum of the cubic ($d = 3$) and quintic ($d = 5$) B-spline filter as a function of a normalized frequency ω/ω_0 with $\omega_0 = \frac{2\pi}{T}$.

In order to guarantee a good tracking of the B-spline function defined by the control points p_k it is necessary that the frequency spectrum of the trajectory is included in the bandwidth of $P(s)$. Since, the B-spline is a linear combination of the B-basis function $B^d(t)$, whose Laplace transform is represented by the chain of filters composing the trajectory generator in Fig. 2, its frequency spectrum can be evaluated by considering the frequency response of such filters, i.e.

$$B^d(j\omega) = \left(\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}} \right)^{d+1} e^{-j\omega \frac{(d+1)}{2} T}$$

The B-spline filter/function is characterized by a pure delay of $\frac{(d+1)T}{2}$ seconds, while the magnitude decreases as $1/\omega^{d+1}$, see Fig. 4 where the frequency spectra for cubic and quintic B-splines are shown. From a practical point of view, spectral components for $\omega \geq \omega_0 = \frac{2\pi}{T}$ can be neglected, in particular for higher values of d. This means that a good tracking can be achieved if $\omega^\star > \omega_0$, where $\omega^\star$ denotes the break frequency of $P(s)$. Finally, is worth noticing that the slope of $|B^d(j\omega)|$ for $\omega \to \infty$ is $-(d+1) \times 20$ db/decade. As a consequence, being $d \geq 2m_u$ (see property 1 in sec. II) the cascade of the trajectory generator and $P(s)$ will have a negative slope for high frequency values and will be always limited in magnitude.

IV. Repetitive Control Based on B-spline Trajectory Generator and ZPETC

In the first part of this section some basic and well-known results of repetitive control are reviewed in order to understand the motivations of the proposed approach, which is illustrated in the second part of the section.

A. *Nyquist-driven stability condition for repetitive control schemes*

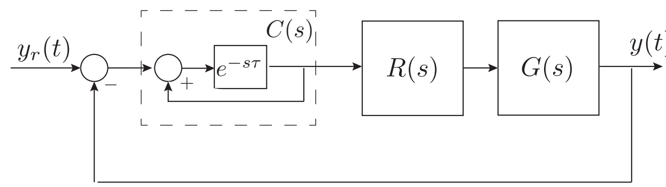

Fig. 5. Basic structure of (continuous) repetitive control.

The basic scheme of Repetitive Control (RC) is shown in Fig. 5. It is composed by:

- a linear time invariant SISO plant $G(s)$;
- an internal model based controller $C(s)$, that guarantees asymptotically zero tracking error of any periodic reference signal of period τ, i.e. $y_r(t+\tau) = y_r(t)$;
- a stabilizing compensator $R(s)$.

The main issue of this control approach consists in the design of $R(s)$ that assures the stability of the loop. Stability condition for the RC scheme can be easily derived by means of classical Nyquist analysis. The characteristic equation of the scheme is

$$1 + \frac{e^{-\tau s}}{1 - e^{-\tau s}} P(s) = 0 \quad (7)$$

where $P(s) = R(s)G(s)$. After some algebraic manipulation, equation (7) can be rewritten as

$$1 + e^{-\tau s}(P(s) - 1) = 0 \quad (8)$$

which can be interpreted as the characteristic equation of the dynamic system shown in Fig. 6, in which the positive feedback loop of $C(s)$ is no more present. By applying Nyquist criterion to the scheme of Fig. 6 it descends that the poles of (7) are stable if and only if the polar plot of the loop function $L(j\omega) = e^{-\tau j\omega}(P(j\omega) - 1)$ does not encircle or

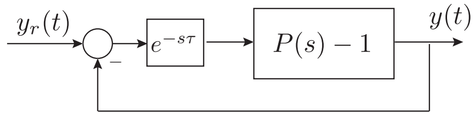

Fig. 6. Equivalent scheme of repetitive control for stability analysis.

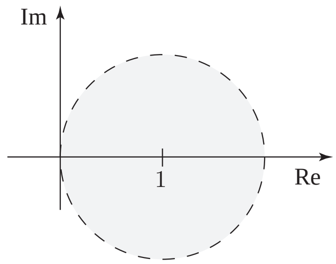

Fig. 7. Stability region of RC for $P(j\omega)$.

touch the critical point -1. This can be assured by imposing that

$$|P(j\omega)-1|<1, \ \ \forall\omega. \tag{9}$$

Note that, according to (9), the repetitive control scheme is stable if the polar plot of $P(j\omega)$ completely lies in the open set defined by a unit circle centred in $1+0j$, see Fig. 7. For this reason, continuous repetitive control cannot by applied to a plant $G(s)$ with relative degree $\gamma>0$. In fact, the polar plot of $P(j\omega)=R(j\omega)G(j\omega)$, whose relative degree cannot be smaller than γ because of the causality of the controller $R(s)$, will always converge to the origin of the complex plane as $\omega\rightarrow\infty$. A solution typically adopted in literature consists in a discrete-time implementation (with a sampling time T_s) of the repetitive control. In this case, the stability of the scheme of Fig. 8 can be assured if a condition analogous to (9) holds but only in the frequency range $[0,\pi/T_s]$.

B. Repetitive control scheme based on B-spline trajectory generator and ZPETC

In order to eliminate the tracking error due to the open-loop structure of ZPETC, the overall system shown in Fig. 3, including the block for the computation of the control points from the desired via-points, has been embedded in an RC scheme, as shown in Fig. 9. Note that this approach, based on the tracking error between the real position $q_k=q(t=kT)$ and the desired position $q_k^\star$ at the interpolation time-instant kT, leads to a discrete-time scheme with sampling time $T_s=T$, i.e. the temporal distance among the desired via-points. This scheme is completely equivalent to the basic structure reported in Fig. 8, if one assumes that

$$P(z)=K_p\cdot H(z)\cdot z^m\cdot\mathcal{Z}\{H_0(s)M^d(s)R_{\text{ZPET}}(s)G(s)\} \tag{10}$$

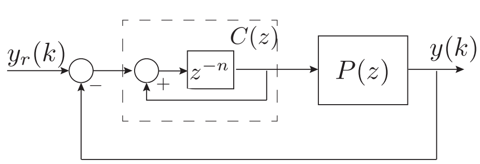

Fig. 8. Discrete-time repetitive control.

where

$$HMRG(z)=\mathcal{Z}\{H_0(s)M^d(s)R_{\text{ZPET}}(s)G(s)\}$$

denotes the z-transform of the continuous time system composed by B-spline generator, ZPET controller and plant, K_p is a proportional gain (usually assumed equal to 1),

$$H(z)=\sum_{n=-r}^{r}h(n)\ z^{-n} \tag{11}$$

is a FIR filter, that approximates the relationship between the via-points $q_k^\star$ and the control points p_k (for more details refer to [9] and [4]). Note that $H(z)$, characterized by $h(n)=h(-n)$, has a frequency response $H(e^{j\omega T})$ which is a positive real function of ω and whose argument is therefore null in the overall frequency range.

The RC scheme guarantees asymptotically a perfect interpolation of the via-points if $P(z)$ complies with (9) in the frequency range $[0,\pi/T]$. In particular, this condition can be met only if

$$\angle P(e^{j\omega T})<\frac{\pi}{2}\text{rad}, \ \ \omega\leq\frac{\pi}{T} \tag{12}$$

and this explains the role of ZPET controller. The property 2 reported in Sec. II, and the frequency response of the trajectory generator along with the zero-order hold guarantee

$$\angle H_0(j\omega)M^d(j\omega)R_{\text{ZPET}}(j\omega)G(j\omega)=-m\omega T.$$

Therefore, as already mentioned in Sec. III the continuous system is characterized by a pure delay of m sample periods T, caused by the trajectory generator. The corresponding discrete-time system $HMRG(z)$ will have the same pure delay, and can be written as

$$HMRG(z)=z^{-m}L(z) \tag{13}$$

where $L(z)$ is a zero-phase filter. In Fig. 10 the typical frequency response of the system $HMRG(z)$ obtained by discretization, with sampling frequency $\omega_0=2\pi/T$, is shown and compared with the original system.

By substituting (13) in (10) the expression

$$P(z)=K_p\cdot H(z)\cdot L(z)$$

is obtained. Therefore, also $P(z)$ is a zero-phase filter. Moreover, since

$$0<|H(e^{j\omega T})|\cdot|L(e^{j\omega T})|<\infty, \ \ \omega\leq\frac{\pi}{T}$$

by acting on K_p is always possible to impose $|P(e^{j\omega T})|\in\,]0,2[$, i.e. inside the stability region. In many circumstances, e.g. when $\omega^\star>\omega_0$, $|H(e^{j\omega T})|\cdot|L(e^{j\omega T})|\approx 1$ in the overall frequency range and consequently $K_p=1$. Note that if the plant is minimum phase and therefore its dynamics is fully cancelled by the ZPET controller $|H(e^{j\omega T})|\cdot|L(e^{j\omega T})|=1$ in the overall frequency range. This is, at least in the nominal case, the most favorable situation since $P(e^{j\omega T})$ is exactly in the center of the stability region illustrated in Fig. 7.

Note that in (10) a time-anticipation z^m appears, but this is

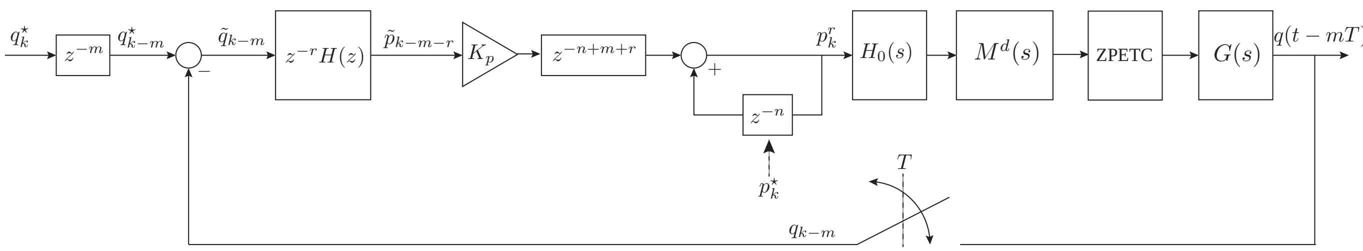

Fig. 9. Discrete-time repetitive control scheme based on discrete-time B-spline filter.

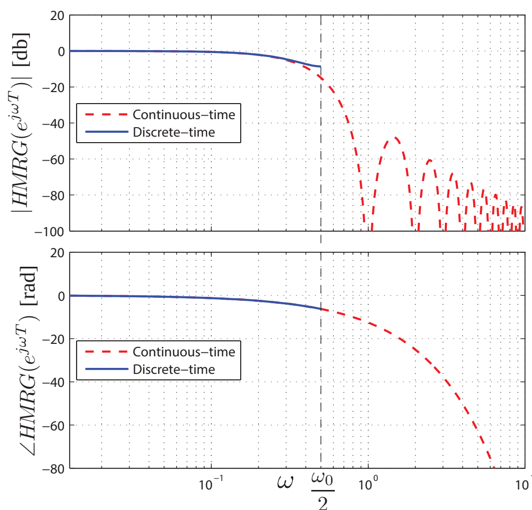

Fig. 10. Frequency response of a discrete-time system $HMRG(z)$ obtained by sampling (with period T) the continuous-time system $H_0(s)M^d(s)R_{\text{ZPET}}(s)G(s)$ with $d = 3$ and $G(s)$ with an unstable zero.

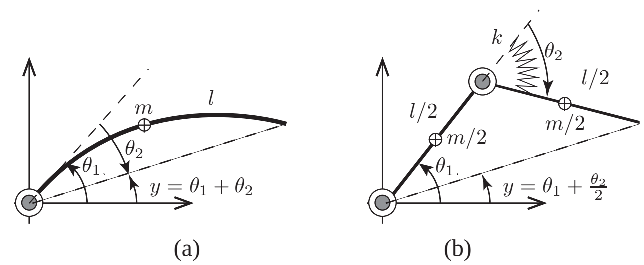

Fig. 11. One-link flexible arm (a) and equivalent model with the deformation concentrated in an elastic hinge (b).

only due to analysis purposes and in the original scheme of Fig. 9 no anticipations are present. Moreover, the zero-phase filter $H(z)$, which is not causal, in this scheme is delayed by r samples in order to make it feasible.

V. Repetitive control of a one link flexible arm

A well-known example of non-minimum phase system is represented by a flexible link. In fact its zero dynamics associated with any reasonable definition of an output is unstable regardless the accuracy of the model, whether it is chosen to be nonlinear or linear, infinite- or finite-dimensional, with any number of elastic modes [10].

A. Flexible Link Model

The system that has been chosen as a case of study is a one-link flexible planar robot arm with mass m and length l, driven by an input torque τ, see Fig. 11(a).

A simple and effective model of this plant can be obtained by considering the total compliance of the link lumped in a hinge located in the middle of the arm, see Fig. 11(b). In this way, only the first elastic mode is considered and the nonlinear dynamic equations are provided in [10].

The tip angular position seen from the base $y = \theta_1 + \frac{\theta_2}{2}$ has been assumed as controlled output and the state vector $x = (\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)^T$ has been considered. With this choice, the state space representation of the system, linearized about $x^\star = (\theta_1^\star, 0, 0, 0)^T$ where $\theta_1^\star$ denotes a generic value of the angular position θ_1, is

$$\dot{x} = \mathbf{A}x + \mathbf{B}\tau$$
$$y = \mathbf{C}x$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b_{22}d_1}{D} & \frac{b_{12}k}{D} & \frac{b_{12}d_2}{D} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b_{12}d_1}{D} & -\frac{b_{11}k}{D} & -\frac{b_{11}d_2}{D} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{b_{22}}{D} \\ 0 \\ -\frac{b_{12}}{D} \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0.5 & 0 \end{bmatrix}$$

with b_{11}, b_{12}, D function of the system parameters.

The ZPET controller has been designed as described in section II. Since the relative degree γ of $G(s)$ is 2, to satisfy (6) it is necessary that the spline order is greater than 4. Therefore quintic B-splines ($d = 5$) have been used.

B. Simulative results

The control scheme shown in Fig. 9 has been implemented in Matlab/Simulink. The ZPET controller has been designed on the basis of the the linearized system, while the non-linear model of the plant has been taken into account in order to introduce some modelling errors. Additionally, some parametric uncertainties on the viscous friction, Coriolis and inertial coefficients have been intentionally introduced in order to show the robustness of the scheme. The nominal parameters of the system are reported in Tab. I.

When the ZPETC, which is a standard feedforward controller, is used in open loop, it exhibits a very poor robustness with respect to modelling errors or wrong initial conditions

Parameter	Symbol	Value	Unit
Link 1 length	$l/2$	0.072	m
Link 2 length	$l/2$	0.072	m
Link 1 mass	$m/2$	0.26	Kg
Link 2 mass	$m/2$	0.26	Kg
Joint 2 stiffness	k	1.5	N/rad
Joint 1 damping	d_1	0.05	N s/rad
Joint 2 damping	d_2	0.01	N s/rad

TABLE I

ONE-LINK FLEXIBLE ARM PARAMETERS

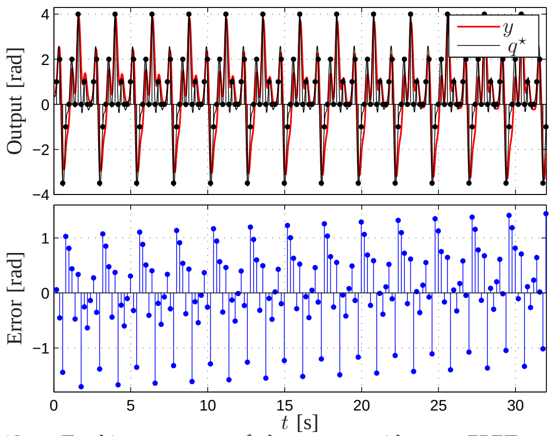

Fig. 12. Tracking response of the system with pure ZPET controller forward compensation without RC.

(see Fig. 12). On the contrary, the use of the Repetitive Control together with ZPETC, guarantees asymptotic perfect tracking at desired via-points, even if the system is affected by parametric uncertainties and/or exogenous periodic disturbances. In Fig. 13 the closed loop response is shown. The initial error due to modelling errors converges to zero after few cycles. In Fig. 14 a time detail of the tenth period, when the interpolation error is practically expired, is shown. It can be notices that the Repetitive Control has modified the control points p_k and consequently the shape of the curve $q^r(t)$ which is actually provided to the the system composed by plant and ZPETC from its initial value $q^\star(t)$. With this new input the plant is able to interpolate perfectly the desired via-points $q_k^\star$.

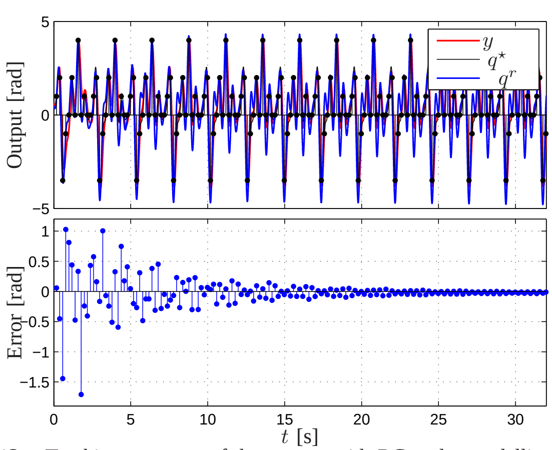

Fig. 13. Tracking response of the system with RC under modelling errors

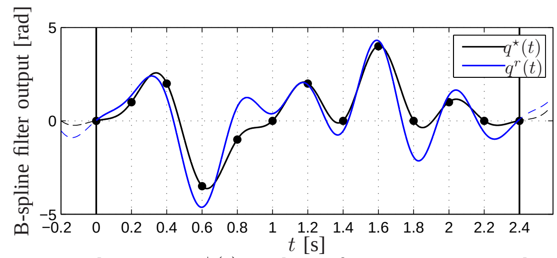

Fig. 14. B-spline curve $q^\star(t)$ used as reference trajectory and trajectory $q^r(t)$ modified by the RC.

VI. CONCLUSIONS

In this paper a novel control scheme able to obtain the perfect asymptotic tracking of a set of desired via-points, used for generating an interpolation B-spline trajectory, has been presented. The proposed approach, based on dynamic filters for B-spline trajectories generation together with a feedforward technique, named ZPET, that can be applied to non-minimum phase systems and the repetitive control scheme, combines the performance of feedforward control in nominal conditions with the robustness of internal model control.

The validity of the overall control scheme has been proved analytically via a stability analysis and by means of simulation on a well-known non-minimum phase system, i.e. a flexible link arm, whose linear model has been obtained by linearization. The extension of the proposed approach, based on RC and feedforward compensation, to nonlinear systems will be considered in the near future.

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